Erratum
Smooth representations of $\mathrm{GL}_m(D)$
V: endo-classes

Kazutoshi Kariyama drawn our attention to the fact that an argument is missing in our proof of [**1**] Proposition 4.5. We cannot use [**1**] Theorem 4.2 at the end of the proof, since we do not know that $(d, f_F(\beta_1)) = (d, f_F(\beta_2))$ at this stage.

In this erratum, we explain why [**1**] Proposition 4.5 can be replaced by the following statement. We use the notation of [**1**].

For $i = 1, 2$, let (k, β_i) be a simple pair over F, let $[\Lambda, n_i, m_i, \varphi_i(\beta_i)]$ be a realization of (k, β_i) in A and let θ_i be a simple character in $\mathcal{C}(\Lambda, m_i, \varphi_i(\beta_i))$.

Proposition 0.1. — *Assume θ_1 and θ_2 intertwine in $A^\times$, and either $[F[\beta_1] : F] = [F[\beta_2] : F]$ or $m_1 = m_2$.*

(1) *We have:*

$$\begin{aligned} n_1 &= n_2, \\ e_F(\beta_1) &= e_F(\beta_2), \\ f_F(\beta_1) &= f_F(\beta_2), \\ k_F(\beta_1) &= k_F(\beta_2). \end{aligned}$$

(2) *There is a simple central F-algebra A' together with realizations $[\Lambda', n_i, m_i, \varphi'_i(\beta_i)]$ of the pairs (k, β_i) in A' (with the same n_i and m_i), with $i = 1, 2$, which are sound and have the same embedding type, and such that θ'_1 and θ'_2 intertwine in $A'^\times$, where $\theta'_i \in \mathcal{C}(\Lambda', m_i, \varphi'_i(\beta_i))$ denotes the transfer of θ_i.*

Proof. — Set $f = (f_F(\beta_1), f_F(\beta_2))$. For $i = 1, 2$, let K_i be the unramified subextension of $F[\beta_i]$ over F of degree f. Using [**1**] Lemma 4.4, we may assume that $(F[\varphi_1(\beta_1)], \Lambda)$ and $(F[\varphi_2(\beta_2)], \Lambda)$ both have Fröhlich invariant 1. The same holds for $(\varphi_1(K_1), \Lambda)$ and $(\varphi_2(K_2), \Lambda)$. Passing to the lattice sequence $\Lambda' = \Lambda^\ddagger$ for a sufficiently large coefficient l, we may, thanks to [**1**] Lemma 4.3, assume that the embeddings $(\varphi'_1(K_1), \Lambda')$ and $(\varphi'_2(K_2), \Lambda')$

– have the same Fröhlich invariant (equal to 1),
– and that they are sound and respectively $\varphi'_1(K_1)$-special and $\varphi'_2(K_2)$-special.

Since they have the same degree f by construction, [**1**] Theorem 4.2 implies that they have the same embedding type. Using the same argument as in the proof of [**2**] 8.4 (or of [**1**] Lemma 4.7), we find that $n_1 = n_2$, denoted n.

Assume that $[F[\beta_1] : F] = [F[\beta_2] : F]$ and $m_1 \geqslant m_2$. Following the proof of [**1**] Lemma 4.7, we get that the stratum $[\Lambda', n, m_1, \varphi'_2(\beta_2)]$ is simple, θ'_1 intertwines with the restriction θ'_0 of θ'_2 to $H^{m_1+1}(\Lambda', \varphi'_2(\beta_2))$ and $e_F(\beta_1) = e_F(\beta_2)$, $f_F(\beta_1) = f_F(\beta_2)$ and $k_F(\beta_1) = k_F(\beta_2)$. Also, θ'_1 is conjugate to θ'_0. Now we know that $[\Lambda', n, m_1, \varphi'_1(\beta_1)]$ and $[\Lambda', n, m_2, \varphi'_2(\beta_2)]$ are sound and have the same embedding type, thanks to [**1**] Theorem 4.2. The fact that θ'_1 and θ'_2 intertwine in $A'^\times$ follows from [**1**] Proposition 2.6.

Assume that $m_1 = m_2$. Applying [**3**] Theorem 10.3 (see [**1**] Theorem 1.16) we get $e_F(\beta_1) = e_F(\beta_2)$ and $f_F(\beta_1) = f_F(\beta_2)$. We thus get the identity $[F[\beta_1] : F] = [F[\beta_2] : F]$ and are reduced to the previous case. □

Remark 0.2. — Lemmas 4.7 and 4.14 of [**1**] are somewhat encapsulated in this new statement Proposition 0.1: the first one uses the assumption $[F[\beta_1] : F] = [F[\beta_2] : F]$, the second one uses the assumption $m_1 = m_2$.

Remark 0.3. — Skodlerack [**4**] Proposition 5.30 fills a gap in the proof of [**3**] Proposition 9.1 on which [**3**] Theorem 10.3 relies: see the comment about it in the proof of [**4**] Proposition 5.31.

References

1. P. Broussous, V. Sécherre and S. Stevens, *Smooth representations of* $\mathrm{GL}_m(D)$*, V: endo-classes*, Documenta Math. 17 (2012), 23–77.
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4. D. Skodlerack, *Semisimple characters for inner forms, I:* $\mathrm{GL}_m(D)$, preprint version available on arxiv `https://arxiv.org/abs/1703.04904v2`.