

T D

2.

exo 1.

$$\begin{aligned}
 \text{Pf: (a) } I_1 &= \int_1^2 \left(x^2 + \frac{3}{x^2}\right) dx \\
 &= \left(\frac{1}{3}x^3 - \frac{3}{x}\right) \Big|_1^2 \\
 &= \frac{1}{3} \times (8-1) - 3 \times \left(\frac{1}{2}-1\right) \\
 &= \frac{7}{3} + \frac{3}{2} \\
 &= \frac{23}{6}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } I_2 &= \int_1^2 (2x - 4e^{3x}) dx \\
 &= \left(2x - \frac{4}{3}e^{3x}\right) \Big|_1^2 \\
 &= 4 - 2 - \frac{4}{3}(e^6 - e^3) \\
 &= 2 - \frac{4}{3}e^6 + \frac{4}{3}e^3
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) } I_3 &= \int_0^1 \frac{x+1}{x^2+2x+5} dx \\
 &= \int_0^1 \frac{1}{2} \cdot \frac{d(x+1)^2}{(x+1)^2+4} \\
 &= \frac{1}{2} \ln((x+1)^2+4) \Big|_0^1 \\
 &= \frac{1}{2} \ln 8 - \frac{1}{2} \ln 5
 \end{aligned}$$

$$\begin{aligned}
 \text{(d) } I_4 &= \int_1^2 \frac{e^x}{x^2} dx \\
 &= \int_1^2 -e^{\frac{1}{x}} d\frac{1}{x} \\
 &= -e^{\frac{1}{x}} \Big|_1^2 \\
 &= -e^{\frac{1}{2}} + e
 \end{aligned}$$

$$\begin{aligned}
 \text{(e) } I_5 &= \int_0^1 (2x+3)\sqrt{x^2+3x+4} dx \\
 &= \int_0^1 d(x^2+3x) \sqrt{x^2+3x+4} \\
 &= \int_0^1 \frac{2}{3} d(x^2+3x+4)^{\frac{3}{2}} \\
 &= \frac{2}{3} (x^2+3x+4)^{\frac{3}{2}} \Big|_0^1 \\
 &= \frac{2}{3} \times 8^{\frac{3}{2}} - \frac{2}{3} \times 4^{\frac{3}{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(f) } I_6 &= \int_0^1 \frac{1}{1+x^2} dx \\
 &= \arctan x \Big|_0^1 \\
 &= \frac{\pi}{4}
 \end{aligned}$$

exo. 2.

$$\begin{aligned}
 \text{Pf: (a) } \int f(x) dx &= \int x e^{x^2} dx \\
 &= \frac{1}{2} \int e^{x^2} dx^2 \\
 &= \frac{1}{2} e^{x^2} + C
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } \int g(x) dx &= \int \frac{x^2}{1+x^3} dx \\
 &= \frac{1}{3} \int \frac{dx^3}{1+x^3} \\
 &= \frac{1}{3} \ln(1+x^3) + C
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) } \int h(x) dx &= \int \frac{\ln x}{x} dx \\
 &= \int \ln x d\ln x \\
 &= \frac{1}{2} (\ln x)^2 + C
 \end{aligned}$$

$$\begin{aligned}
 (d) \int i(x) dx &= \int \frac{x}{\sqrt{1+x^2}} dx \\
 &= \frac{1}{2} \int \frac{dx^2}{\sqrt{1+x^2}} \\
 &= (1+x^2)^{\frac{1}{2}} + C
 \end{aligned}$$

$$\begin{aligned}
 (e) \int j(x) dx &= \int \frac{1}{x \ln x} dx \\
 &= \int \frac{d \ln x}{\ln x} \\
 &= \begin{cases} \ln(\ln x) + C & x > 1 \\ \ln(-\ln x) + C & 0 < x < 1 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 (f) \int k(x) dx &= \int \tan x dx \\
 &= \int \frac{\sin x}{\cos x} dx \\
 &= \int -\frac{d \cos x}{\cos x} \\
 &= -\ln |\cos x| + C
 \end{aligned}$$

exo 3.

$$\begin{aligned}
 \text{pf: (a)} \quad I_1 &= \int_1^e \ln x dx \\
 &= x \ln x \Big|_1^e - \int_1^e x \cdot \frac{1}{x} dx \\
 &= e - \int_1^e dx \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad I_2 &= \int_2^3 \frac{x}{\sqrt{x-1}} dx \\
 &= \int_2^3 \sqrt{x-1} dx + \int_2^3 \frac{1}{\sqrt{x-1}} dx \\
 &= \frac{2}{3} (x-1)^{\frac{3}{2}} \Big|_2^3 + 2(x-1)^{\frac{1}{2}} \Big|_2^3 \\
 &= \frac{2}{3} \times 2^{\frac{3}{2}} - \frac{2}{3} + 2 \times 2^{\frac{1}{2}} - 2 \\
 &= \frac{16\sqrt{2}}{3} - \frac{8}{3}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad I_3 &= \int_e^{2e} x^2 \ln x dx \\
 &= \frac{1}{3} x^3 \ln x \Big|_e^{2e} - \int_e^{2e} \frac{1}{3} x^3 \cdot \frac{1}{x} dx \\
 &= \frac{1}{3} x^3 \ln x \Big|_e^{2e} - \frac{1}{9} x^3 \Big|_e^{2e} \\
 &= \frac{8}{3} e^3 \ln 2 + \frac{8}{3} e^3 - \frac{e^3}{3} - \frac{8}{9} e^3 + \frac{1}{9} e^3 \\
 &= \frac{8}{3} e^3 \ln 2 + \frac{7}{3} e^3 - \frac{7}{9} e^3 \\
 &= \frac{8}{3} e^3 \ln 2 + \frac{14}{9} e^3
 \end{aligned}$$

$$\begin{aligned}
 (d) \quad I_4 &= \int_{-1}^0 (-2x+1) e^{-x} dx \\
 &= -e^{-x} (-2x+1) \Big|_{-1}^0 - \int_{-1}^0 -2(-e^{-x}) dx \\
 &= 3e - 1 - \int_{-1}^0 2e^{-x} dx \\
 &= 3e - 1 + 2e^{-x} \Big|_{-1}^0 \\
 &= 3e - 1 + 2 - 2e \\
 &= e + 1
 \end{aligned}$$

$$\begin{aligned}
 (e) \quad I_5 &= \int_1^e (\ln x)^2 dx \\
 &= x (\ln x)^2 \Big|_1^e - \int_1^e 2 \ln x \cdot \frac{1}{x} \cdot x dx \\
 &= e - \int_1^e 2 \ln x dx \\
 &= e - 2x \ln x \Big|_1^e + \int_1^e \frac{2}{x} \cdot x dx \\
 &= e - 2e + 2x \Big|_1^e \\
 &= -e + 2e - 2 \\
 &= e - 2
 \end{aligned}$$

exo 4.

$$\begin{aligned}
 (f) I_6 &= \int_0^1 x \arctan x \, dx \\
 &= x \arctan x \Big|_0^1 - \int_0^1 \frac{x}{x^2+1} \, dx \\
 &= \frac{\pi}{4} - \frac{1}{2} \ln(x^2+1) \Big|_0^1 \\
 &= \frac{\pi}{4} - \frac{1}{2} \ln 2
 \end{aligned}$$

$$\begin{aligned}
 \text{Pf: 1. (a)} \int_1^3 \frac{dx}{\sqrt{x} + \sqrt{x^3}} \\
 &= \int_1^3 \frac{2 \, d\sqrt{x}}{1 + (\sqrt{x})^2} \\
 &= 2 \arctan \sqrt{x} \Big|_1^3 \\
 &= 2 \arctan \sqrt{3} - 2 \arctan 1 \\
 &= 2 \times \frac{\pi}{3} - 2 \times \frac{\pi}{4} \\
 &= \frac{\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 (g) I_7 &= \int_0^{\frac{\pi}{2}} e^x \cos x \, dx \\
 &= e^x \sin x \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \sin x \, dx \\
 &= -1 + e^{\frac{\pi}{2}} \sin \frac{\pi}{2} + \int_0^{\frac{\pi}{2}} e^x (-\cos x) \, dx \\
 &= -1 + e^{\frac{\pi}{2}} - I_7
 \end{aligned}$$

$$\text{Ahrs } I_7 = \frac{-1 + e^{\frac{\pi}{2}}}{2}$$

$$\begin{aligned}
 (b) \int_1^{e^2} \frac{\ln x}{x + x(\ln x)^2} \, dx \\
 &= \int_1^{e^2} \frac{\ln x}{1 + (\ln x)^2} \, d \ln x \\
 &= \int_1^{e^2} \frac{1}{2} \cdot \frac{d(\ln x)^2}{1 + (\ln x)^2} \\
 &= \frac{1}{2} \ln(1 + (\ln x)^2) \Big|_1^{e^2} \\
 &= \frac{1}{2} \ln 5
 \end{aligned}$$

$$\begin{aligned}
 (h) I_8 &= \int_0^1 \ln(1+x^2) \, dx \\
 &= x \ln(1+x^2) \Big|_0^1 - \int_0^1 x \cdot (2x) \cdot \frac{1}{1+x^2} \, dx \\
 &= \ln 2 - 2 \int_0^1 \frac{dx}{1+x^2} + \int_0^1 \frac{2}{1+x^2} \, dx \\
 &= \ln 2 - 2 + 2 \arctan x \Big|_0^1 \\
 &= \ln 2 - 2 + \frac{\pi}{2}
 \end{aligned}$$

$$\begin{aligned}
 (c) \int_0^1 \frac{e^{2x}}{e^x+1} \, dx \\
 &= \int_0^1 \frac{e^{2x}}{e^x+e^x} \, dx
 \end{aligned}$$

~~u/lor~~

$$\begin{aligned}
 &= \int_0^1 \frac{e^x}{1+e^x} \, dx \\
 &= \int_0^1 dx - \int_0^1 \frac{dx}{1+e^x} \\
 &= e^x \Big|_0^1 - \ln(1+e^x) \Big|_0^1 \\
 &= e-1 - \ln(1+e) + \ln 2
 \end{aligned}$$

$$\begin{aligned}
 (i) I_9 &= \int_1^e (x^2+x+2) \ln(x) \, dx \\
 &= \left( \frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \right) \ln(x) \Big|_1^e - \int_1^e \left( \frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \right) \cdot \frac{dx}{x} \\
 &= \frac{1}{3}e^3 + \frac{1}{2}e^2 + 2e - \left( \frac{1}{9}x^3 + \frac{1}{4}x^2 + 2x \right) \Big|_1^e \\
 &= \frac{1}{3}e^3 + \frac{1}{2}e^2 + 2e - \frac{1}{9}e^3 - \frac{1}{4}e^2 - 2e + \frac{1}{9} + \frac{1}{4} + 2 \\
 &= \frac{2}{9}e^3 + \frac{1}{4}e^2 + \frac{85}{36}
 \end{aligned}$$

exo. 5.

Pf:  $q_1(x) = \frac{1}{x^2+x-1}$

$$q_2(x) = \frac{x^2}{(x-2)(x-3)} = x^2 \left( \frac{1}{x-2} - \frac{1}{x-3} \right) (-1)$$

$$= -(x+2 + \frac{4}{x-2} - (x+3) - \frac{9}{x-3}) (-1)$$

$$= +1 + \frac{4}{x-2} + \frac{9}{x-3} = 1 - \frac{4}{x-2} + \frac{9}{x-3}$$

$$q_3(x) = \frac{2x-1}{x(x-1)^2} = \frac{A}{x} + \frac{Bx+C}{(x-1)^2}$$

$$= \frac{(B+A)x^2 + (C-2A)x + A}{x(x-1)^2}$$

Avs  $A+B=0$   $C-2A=2$   $A=-1$

Avs  $A=-1$   $B=1$   $C=0$

$$q_3(x) = \frac{x}{(x-1)^2} - \frac{1}{x}$$

$$q_4(x) = \frac{x^2+1}{x^2-1} = \frac{x^2-x^5+x^5-x^3+x^3-x+x+1}{x^2-1}$$

$$= x^5+x^3+x + \frac{1}{x-1}$$

$$q_5(x) = \frac{5x^2-2x+3}{(x^2+1)(x-1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$= \frac{(A+B)x^2 - Bx + A - C}{(x^2+1)(x-1)}$$

Avs  $A+B=5$   $-B=-2$   
 $A-C=3$

Avs  $A=3$   $B=2$   $C=0$

$$q_5(x) = \frac{3}{x-1} + \frac{2x}{x^2+1}$$

$$(d) \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos^4 x} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{-d\cos x}{\cos^4 x}$$

$$= \frac{1}{3} \cdot \frac{1}{\cos^3 x} \Big|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{3} (\sqrt{2})^3 - \frac{1}{3}$$

$$2. (a) \int \frac{\sin x \cos x}{1-\cos x} dx$$

$$= \int \frac{\cos x}{1-\cos x} d(-\cos x)$$

$$= \int d\cos x + \int \frac{d\cos x}{\cos x - 1}$$

$$= \cos x + \ln|\cos x - 1| + C$$

$$= \cos x + \ln(1 - \cos x) + C$$

$$(b) \int \frac{dx}{\cos x} = \int \frac{\cos x dx}{\cos^2 x}$$

$$= \int \frac{d\sin x}{1-\sin^2 x}$$

$$= \frac{1}{2} \int \frac{d\sin x}{1-\sin x} + \frac{1}{2} \int \frac{d\sin x}{1+\sin x}$$

$$= -\frac{1}{2} \ln(1-\sin x) + \frac{1}{2} \ln(1+\sin x) + C$$

$$(c) \int \sqrt{e^x-1} dx$$

$$= \int \frac{e^x-1}{e^x} dx$$

$$\underline{u=e^x-1} \int \frac{u du}{u^2+1}$$

$$= \int \frac{2u^2 du}{u^2+1}$$

$$= 2 \int du - 2 \int \frac{du}{u^2+1}$$

$$= 2u - 2 \arctan u + C$$

$$= 2\sqrt{e^x-1} - 2 \arctan \sqrt{e^x-1} + C$$