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Convergence rate in the law of large numbers for martingale differences

Quansheng Liu

Laboratoire de Mathématiques de Bretagne Atlantique (CNRS UMR 6205)

Univ. de Bretagne - Sud

We show convergence rates in the Marcinkiewicz laws of large numbers for arrays of martingale differences. For $n \geq 1$, let X_{n1}, X_{n2} $\cdots$, be a sequence of real valued martingale differences with respect to a filtration $\{\emptyset, \Omega\} = \mathcal{F}_{\backslash\prime} \subset \mathcal{F}_{\backslash\infty} \subset \mathcal{F}_{\backslash\in} \subset \cdots$, and set $S_{nn} = X_{n1} + \cdots + X_{nn}$. Under a simple moment condition on $\sum_{j=1}^{n} E[|X_{nj}|^{\gamma}|\mathcal{F}_{\backslash,|-\infty}]$ for some $\gamma \in (1, 2]$, we show criterions for the convergence of the series $\sum_{n=1}^{\infty} \phi(n)P\{|S_{nn}| > \varepsilon n^{\alpha}\}$, where $\alpha > 0$, ϕ is a positive function and $\varepsilon > 0$; we also give a criterion for $\phi(n)P\{|S_{nn}| > \varepsilon n^{\alpha}\} \to 0$. The most interesting case where ϕ is a regularly varying function is considered with attention. In the special case where $(X_{nj})_{j\geq1}$ are the same sequence $(X_j)_{j\geq1}$ of independent and identically distributed random variables, our result on the series $\sum_{n=1}^{\infty} \phi(n)P\{|S_{nn}| > \varepsilon n^{\alpha}\}$ contains the theorems of Hsu-Robbins-Erdös (1947, 1949, 1950) if $\alpha = 1$ and $\phi(n) = 1$, of Spitzer (1956) if $\alpha = 1$ and $\phi(n) = 1/n$, and of Baum and Katz (1965) if $\phi(n) = n^{b-1}$ with $b \geq 0$. In the single martingale case (where $X_{nj} = X_j$ for all n and j), it generalizes the results of Alsmeyer (1990), and completes those of Lesigne and Volný (2001). The consideration of martingale arrays (rather than a single martingale) makes the results very adapted in the study of weighted sums of identically distributed random variables, for which we prove new theorems about the rates of convergence in the law of large numbers. The results are established in a more general setting for sums of infinite many martingale differences X_{nj}, say $S_{n,\infty} = \sum_{j=1}^{\infty} X_{nj}$ instead of S_{nn}. The obtained results improve and extend those of Ghosal and Chandra (1998). The one-sided cases and the supermartingale case are also considered.