EXACT FRACTIONAL QUANTUM HALL STATES

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Outline

- some key facts about the FQHE : the wavefunction approach
- Find a quasi-solvable limit for the FQHE which is not simple electrostatics
- exhibit infinitely many exact eigenstates that are simple in second-quantized language

work with Paul Soulé : arXiv:1111.2519

FQHE

Fractional quantum Hall liquids :

- one needs particles (Fermi or Bose) in 2D plane with a strong magnetic field
- realized in 2D semiconductors heterostructures
- maybe one day in rotating atomic gases where Coriolis force can mimic Lorentz force

FQHE

- The 2D situation under a field has no kinetic energy : instead highly degenerate Landau levels.
- Only interactions fix the nature of the ground state.
- For many rational fillings $\nu = p/q$ of the lowest Landau level, the ground state is a liquid with *gapped* excitations.
- Quasiholes and quasielectrons with fractional charge and statistics.

Laughlin's *tour de force*

- guess wavefunctions describing the ground state and some key excited states :

$$\Psi_{GS} = \prod_{i<j} (z_i - z_i)^3 \; \mathrm{e}^{-\sum_i |z_i|^2/4} \, \text{for } \nu = 1/3$$

$$\Psi_{QH} = \prod_i (z_i - Z_0)\Psi_{GS}, \quad \Psi_{QE} = \prod_i (\frac{\partial}{\partial z_i} - Z_0)\Psi_{GS}$$

- solving the Hamiltonian is *evil*.

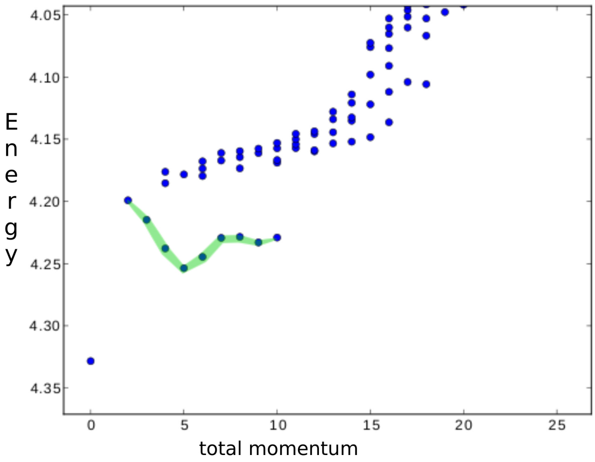
interactions

We need two truncations. The first one is on the range of the interaction. For the interaction we take the extreme repulsive hard-core limit :

- For spinless bosons : $\delta^{(2)}(z_i - z_j)$
- For spinless fermions : $\Delta\delta^{(2)}(z_i - z_j)$

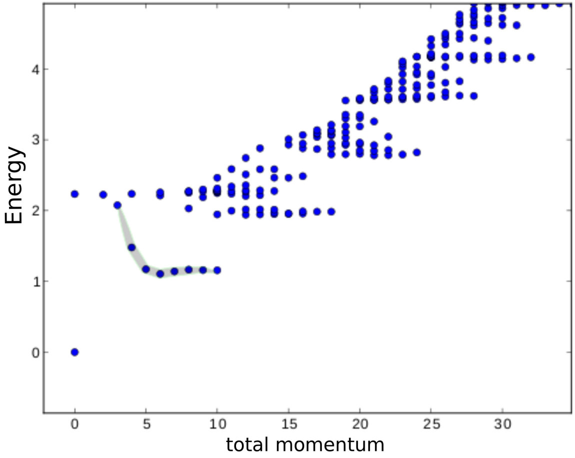
These are known to lead to the FQHE physics from numerical studies.

N=10 electrons with Coulomb interactions in the LLL at filling $\nu = 1/3$



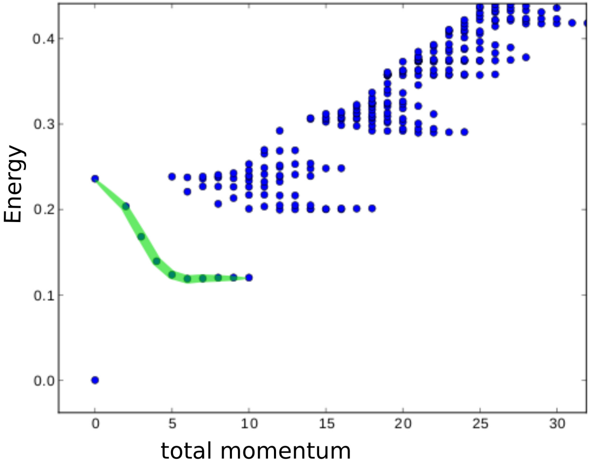
nothing exact is known

N=10 electrons with hard-core interactions in the LLL at filling $\nu = 1/3$



the Laughlin wavefunction is the exact ground state - does not extend to excited states

N=10 Bosons with delta interactions in the LLL at filling $\nu = 1/2$



the Laughlin wavefunction is the exact ground state - does not extend to excited states

Cylinder geometry

We use the Landau gauge and work on a cylinder : with $A_x = 0$ and $A_y = Bx$ The LLL one-body wavefunctions are given by :

$$\phi_n(x, y) = \mathrm{e}^{-(x - k\ell^2)^2/2\ell^2} \mathrm{e}^{iky} = Z^n \ \lambda^{-n^2} \ \mathrm{e}^{-x^2/2\ell^2},$$

$$\lambda = \exp(-2\pi^2\ell^2/L^2), \quad Z \equiv \mathrm{e}^{\frac{2\pi}{L}(x+iy)}, \quad \ell = \sqrt{\hbar c/eB}$$

The Hilbert space is truncated by imposing $|n| \leq N_{max}$ The momentum k is then quantized : $k = 2\pi n/L$ where n is a positive or negative integer.

Cylinder truncation

Write the Hamiltonian for hard-core interactions in second-quantized language :

$$\mathcal{H} = \sum_{n_1+n_2=n_3+n_4} [(n_1-n_3)^2-(n_1-n_4)^2] \quad \lambda^{(n_1-n_3)^2+(n_1-n_4)^2} \quad a^\dagger_{n_1} a^\dagger_{n_2} a_{n_3} a_{n_4} \,,$$

This is for fermions. Remove the $[\ldots]$ for bosons.

$$\mathcal{H} = \sum_{p\geq 0} \lambda^p \mathcal{H}_p, \quad \lambda = \exp(-2\pi^2\ell^2/L^2).$$

$L \to 0, \lambda \to 0$ is the Tao-Thouless Thin-Torus limit. Our second truncation is in this series in powers of λ.

Truncated Hamiltonians

$$\begin{aligned}\mathcal{H}_9^{Fermi} &= \lambda \sum_i n_i n_{i+1} + 4\lambda^4 \sum_i n_i n_{i+2} + 9\lambda^9 \sum_i n_i n_{i+3} \\ &\quad -3\lambda^5 \left[\sum_i c_i^\dagger c_{i+1} c_{i+2} c_{i+3}^\dagger + h.c. \right].\end{aligned}$$

The corresponding Bose Hamiltonian is given by :

$$\begin{aligned}\mathcal{H}_4^{Bose} &= \sum_i n_i(n_i - 1) + 4\lambda \sum_i n_i n_{i+1} + 4\lambda^4 \sum_i n_i n_{i+2} \\ &\quad +2\lambda^2 \left[\sum_i b_i^\dagger b_{i+1}^2 b_{i+2}^\dagger + h.c. \right].\end{aligned}$$

Factorization

$$
\begin{aligned}
\mathcal{H}_9^{Fermi} = \; & \lambda \sum_i C_i^\dagger C_i + 4\lambda^4 \sum_i n_i n_{i+2} \\
+ \; & \lambda \; n_{-N_{max}} n_{1-N_{max}} + \lambda n_{N_{max}} n_{N_{max}-1},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{H}_4^{Bose} = \sum_i B_i^\dagger B_i \; & + \; 4\lambda \sum_i n_i n_{i+1} \\
+ n_{-N_{max}}(n_{-N_{max}} - 1) \; & + \; n_{N_{max}}(n_{N_{max}} - 1),
\end{aligned}
$$

where we have defined :

$$
C_i = c_{i+2} c_{i+1} + 3\lambda^4 c_{i+3} c_i, \quad B_i = b_{i+1}^2 + 2\lambda^2 b_i b_{i+2}. \tag{1}
$$

We have found an infinite number of exact eigenstates of the truncated models that are of the form :

$$\Psi = \mathcal{S}|\mathrm{root}\rangle$$

where $\mathcal{S}$ is a *squeezing* operator defined by :

$$\begin{aligned}
\mathcal{S}_F &= \prod_n (1 + 3\lambda^4 c_{n-1} c_n^\dagger c_{n+1}^\dagger c_{n+2}), \\
\mathcal{S}_B &= \prod_n (1 - \lambda^2 b_{n-1} (b_n^\dagger)^2 b_{n+1})
\end{aligned}$$

they key point is that $C_n\Psi = 0$ or $B_n\Psi = 0$ for all n.

They are simple in second-quantized language

Squeezing

The squeezing operation on Fock space vectors :

$$\cdots \overrightarrow{1}0\overleftarrow{0}1 \ldots \quad \xrightarrow{\mathcal{S}_F} \quad \ldots 0110 \ldots$$

and for bosons :

$$\cdots \overrightarrow{1}\ 0\ \overleftarrow{1} \ldots \quad \xrightarrow{\mathcal{S}_B} \quad \ldots 020 \ldots$$

This operation generates descendents from the root configuration. This is the dominance order of symmetric polynomials in the Bose case. The squeezings are suppressed in the TT limit $\lambda \to 0$.

Ground state

For the fermionic $\nu = 1/3$ filling, the ground state is given by :

$$\Psi = \mathcal{S}_F|100100100100\ldots1001\rangle$$

In the Bose case :

$$\Psi = \mathcal{S}_B|10101010\ldots101\rangle$$

The root configuration is the electrostatic ground state of the TT limit. They have zero-energy as the Laughlin wavefunction.

Quasielectron

Excited states are also simple (some of them) :

$$\begin{aligned} \Psi_{QE} &= \mathcal{S}|11000100100\ldots001\rangle(\mathrm{Fermi}), \\ \Psi_{QE} &= \mathcal{S}|2001010\ldots01\rangle(\mathrm{Bose}) \end{aligned}$$

nonzero energy due to boundary terms : Hubbard-like repulsion creates the gap. The counting is exactly given by the Composite Fermion theory or fractional exclusion statistics unlike the TT limit.

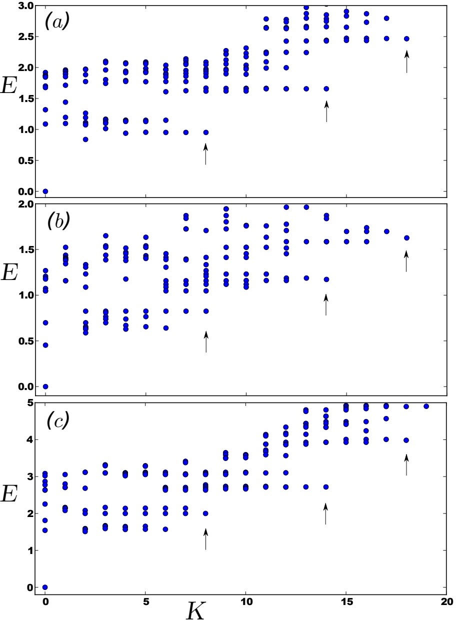
Magnetoroton

We can construct the magnetoroton of Girvin, MacDonald and Platzman :

$$\begin{aligned} \Psi_{MR} &= \mathcal{S}|11000100100\ldots0010\rangle(\mathrm{Fermi}), \\ &= \mathcal{S}|2001010\ldots010\rangle(\mathrm{Bose}) \end{aligned}$$

This is one quasielectron and one quasihole at opposite ends of the cylinder. The energy is λ (resp 1). It is possible to pile up elementary excitations. Not all states are simple : not an exactly solvable model. Akin to the AKLT spin chain, probably.

There is evidence that the truncated model has the right physics.



(a) N=8 electrons at $\nu = 1/3$ with Coulomb interactions

(b) same with truncated Hamiltonian, $\lambda = 0.8$

(c) Bose case at $\nu = 1/2$, $\lambda = 0.7$

Summary

- We have found an almost-solvable limit of the FQHE problem
- Simple in second-quantized language
- contains states of physical interest with the correct counting
- quasiholes have the correct counting but entanglement is trivial
- the TT limit on the cylinder is NOT smooth so we are not solving electrostatics.
- does not work for hierarchical fractions.