Maths-Physics Meeting
around Bose Einstein condensates

November 30th-December 2nd 2011

Institut Henri Poincaré
11 rue Pierre et Marie Curie, Paris, (amphi Darboux)

Wednesday November 30th

9:00 - 9:45

Alexander L. Fetter (Standord, USA) :
Free-expansion of turbulent Bose-Einstein condensate

A Lagrangian variational approach provides an appealing physical picture of the free expansion of a trapped turbulent Bose-Einstein condensate (BEC). A nonrotating vortex-free trapped BEC expands with a reversal of aspect ratio, which is well understood as arising from the combined e-ect of the uncertainty principle and the interparticle repulsion. In contrast, a turbulent condensate has a random set of quantized vortex lines with a line density L per unit volume. For such a condensate, the equations of motion have an additional term that leads asymptotically to a self-similar (scale-invariant) expansion, in qualitative agreementwith experimental observations.

Reference : arXiv:1103:2039v1

10:00 - 10:45

Makota Tsubota (Osaka, Japan)
Quantum Turbulence in Atomic Bose-Einstein Condensates

We discuss recent developments in quantum fluid dynamics and quantum turbulence (QT) in atomic Bose-Einstein condensates. Quantum turbulence was discovered in superfluid He^4 in the 1950s, but this is still one of the most important topics in low temperature physics [1]. The realization of atomic Bose-Einstein condensation in 1995 has proposed another important stage for this issue. Quantum turbulence is comprised of quantized vortices that are definite topological defects arising from the order parameter appearing in Bose-Einstein condensation. Hence QT is expected to yield a simpler model of turbulence than does conventional turbulence. A general introduction to this issue and a brief review of the basic concepts are followed by the recent developments of the studies of QT in atomic Bose-Einstein condensates. I will discuss some of the interesting topics. (1) QT in a trapped BEC created by two precessions.[2]. (2) Nonlinear dynamics of vortices and solitons in a trapped BEC induced by an oscillating potential (Fig.1) [3]. (3) Quantum Kelvin-Helmholtz instability in phase-separated two-component BECs [4]. (4) Binary QT in miscible two-component BECs (Fig. 2) [5].

References :

[1] Progress of Low Temperature Physics Vol.16, eds. W. P. Halperin and M. Tsubota (Elsevier, 2008).

[2] M. Kobayashi, M. Tsubota, Phys. Rev. A76, 045603(2007).

[3] K. Fujimoto, M. Tsubota, Phys. Rev. A 83, 053609(2011).

[4] H. Takeuchi, N. Suzuki, K. Kasamatsu, H. Saito, M. Tsubota, Phys. Rev. B81, 094517(2010).

[5] H. Takeuchi, S. Ishino, M. Tsubota, Phys. Lev. Lett. 105, 205301(2010).

11:15 - 12:00

Clément Gallo (Montpellier, France)

Eigenvalues of a nonlinear ground state in the Thomas-Fermi approximation

This talk will be devoted to the study of the nonlinear ground state of the following cubic-defocusing nonlinear Schrödinger equation with a parabolic potential :

$$iu_t + \varepsilon^2 u_{xx} + (1 - x^2)u|u|^2 u = 0; (t, x) \in \mathbb{R}^2$$

where $\varepsilon > 0$ is a small parameter. This equation arises in the study of Bose-Einstein condensates. Our goal is to describe the oscillations of small perturbations of the ground state, so we study the limit as $\varepsilon \to 0$ of the eigenvalues of the operator obtained by linearization of the equation around the ground state, which will provide the frequencies of these oscillations. In a simplified version of the problem (where the ground state in the expression of the linearized operator is replaced by the Thomas-Fermi approximation), we prove the convergence of the eigenvalues to a sequence of explicit numbers. In order to address the same problem for the true linearized operator, we need a precise description of the behaviour of the ground state, as $\varepsilon \to 0$. We give a full asymptotic expansion of the ground state as $\varepsilon \to 0$, involving solutions of the Painlevé equation. Finally, we show a formal computation that indicates what should be the limit of the eigenvalues of the true linearized operator as $\varepsilon \to 0$.

References :

[1] C. Gallo, D. Pelinovsky, Eigenvalues of a nonlinear ground state in theThomas-Fermi approximation. J. Math. Anal. Appl. 355 (2009), no. 2, 495-526

[2] C. Gallo, D. Pelinovsky, On the Thomas-Fermi ground state in a harmonic potential. Asymptotic Analysis 73 (2011) 53-96

13:30 - 14:15

Franco Dalfovo (Trento, Italia)

Dynamics of dark solitons in superfluid Fermi gases

The dynamics of solitons in a superfluid Fermi gas across the BEC-BCS crossover is investigated by means of both stationary and time-dependent Bogoliubov-de Gennes equations. First we show the results for the oscillations of a dark soliton in a harmonic trap. We derive an exact equation relating the phase jump across the soliton to its energy, and hence obtain an expression for the soliton period. Our analytic approach is supported by simulations of the time-dependent BdG equations, which show that the period dramatically increases as the soliton becomes shallower on the BCS side of the resonance. Then we study soliton collisions and the decay of solitons into sound. This decay process occurs when the solitons are accelerated to the bulk pair-breaking speed by an external potential. A similar decay process may occur when solitons are accelerated by an inelastic collision with another soliton. We find that soliton collisions become increasingly inelastic as we move from the BEC to BCS regimes, and the excess energy is converted into sound. We interpret this effect as due to motion of Andreev bound states localized within the soliton.

14:30 - 15:15

Yvan Martel (Versailles, France)

Collision of solitons for nonintegrable generalized Korteweg de Vries equations

We shall review a series of recent works in collaboration with Frank Merle (Université de Cergy-Pontoise and IHES) concerning the interaction of two solitons for the generalized Korteweg de Vries equations in particular with a quartic nonlinearity. In this case, the equation is nonintegrable and it cannot be solved by the inverse scattering transform.

In two specific asymptotic cases (almost equal speeds / very different speeds), we can describe the collision into details by explicit computations and perturbation theory. We prove that at the main order, the two solitons are preserved by the interaction as in the integrable case. However, unlike in the integrable case, we prove that the collision is inelastic.

References :

Y. Martel and F. Merle, Description of two soliton collision for the quartic gKdV equation, to appear in Annals of Math. http ://arxiv.org/abs/0709.2672

Y. Martel and F. Merle, Inelastic interaction of nearly equal solitons for the quartic gKdV equation, to appear in Inventiones Mathematicae. http://arxiv.org/abs/0910.3204

Thursday December 1st

9:00 - 9:45

Amandine Aftalion (Versailles, France)

Two component condensates : phase separation, vortices and skyrmions

The aim of this talk is to describe the main features arising in the description of a rotating two component condensate. I will show numerical simulations which allow to classify the ground states (regions of coexistence of the components, phase separation and symmetry breaking) and the types of defects. Analytical justifications of some phenomena will be made thanks to a nonlinear sigma model describing the condensate in terms of the total density and pseudo spin representation. In particular, we will estimate the total density in the Thomas fermi regime, both in the case of coexistence of the phases and separations leading either to droplets or annulus plus disk. We will also derive a vortex energy that discriminates between the square and triangular vortex lattices. Finally, we will describe relations of this problem with De Giorgi type problems arising in elliptic PDE's.

10:00 - 10:45

Norman Dancer (Sydney, Autralia)

Some Systems of Nonlinear Elliptic Equations

We discuss the existence of non-negative solutions of some nonlinear elliptic systems of equations. In paricular,we are interested in solutions with all components strictly positive. We also discuss when there is an apriori bound for solutiona and when there is not. This leads to the study of problems on half spaces or the whole space.

11:15 - 12:00

Muneto Nitta (Keio, Japan)

Interaction of Vortices in BEC and QCD

We discuss the interaction between vortices in multi-component systems such as two component Bose-Einstein condensates (BEC) and quantum chromodynamics (QCD). We show that the leading order of the force between two vortices in different components at distance $2R$ in two component BEC is $(logR/\xi - 1/2)/R^3$, in contrast to $1/R$ between two vortices placed in the same component or two vortices in single component BEC. We compare it with the force between vortices in QCD in two different phases. One is the color-flavor locked phase of color superconductivity at extremely high density, in which non-Abelian semi-superfluid vortices (color flux tubes) exist [2], and the other is at chiral phase transition in high temperature, in which non-Abelian global vortices exist [3].

[1] M.Eto, K.Kasamatsu, M.Nitta, H.Takeuchi, M.Tsubota, Phys.Rev.A83:063603,2011

[2] E.Nakano, M.Nitta, T.Matsuura, Phys.Rev.D78:045002,2008

[3] E.Nakano, M.Nitta, T.Matsuura, Phys.Lett.B672:61-64,2009

13:30 - 14:15

Thierry Jolicoeur (Orsay, France)

On some exact fractional quantum hall states

We show how to construct a solvable problem for the fractional quantum Hall states in the bosonic and fermionic cases. While usual description are based on first-quantized wavefunction descriptions, we give exact eigenstates in the language of second quantization. It is then possible to describe the one-quasielectron as well as the magnetoroton excitations in an explicit way.

14:30 - 15:15

Natasha Berloff (Cambridge, UK)

Vortices and solitary waves in nonequilibrium condensates

Spontaneous emergence of spatio-temporal order pattern formation in nonequilibrium systems represents one of the key mechanisms of self-organisation in nature. Similarly, injection and decay of particles in an inhomogeneous quantum condensate can significantly change its behaviour. We model pumped and decaying condensates by a complex Gross-Pitaevskii equation (a complex Ginzburg-Landau equation) and analyse the density and currents in the steady state, vortex solutions and solitary wave dynamics in these non-equilibrium condensates. The new effects arise as from a complex interplay between dispersion, dissipation, forcing and nonlinear interactions. Such nonequilibrium condensates can live in a non-trivial potential landscape, either due to intrinsic disorder in the material, or due to deliberately designed potentials. These potentials may induce stabilising (or destabilising) currents further facilitating symmetry breaking in the system.

15:30 - 15:50

Sylvain Nascimbene (Paris, France)

Experimental realization of strong effective magnetic fields in an optical lattice

We use Raman-assisted tunneling in an optical superlattice to generate large tunable effective magnetic fields for ultracold atoms. When hopping in the lattice, the accumulated phase shift by an atom is equivalent to the Aharonov-Bohm phase of a charged particle exposed to a staggered magnetic field of large magnitude, on the order of one flux quantum per plaquette. We study the ground state of this system and observe that the frustration induced by the magnetic field can lead to a degenerate ground state for non-interacting particles. We provide a measurement of the local phase acquired from Raman-induced tunneling, demonstrating time-reversal symmetry breaking of the underlying Hamiltonian. Furthermore, the quantum cyclotron orbit of single atoms in the lattice exposed to the magnetic field is directly revealed.

Poster session

16:00 - 16:20

Hannah Price (Cambridge, UK)

Skyrmion-antiSkyrmion pairs in Ultracold Atomic Gases

We study theoretically the dynamics of two-component Bose-Einstein condensates in two dimensions, which admit topological excitations related to the Skyrmions of nuclear physics. We investigate a branch of uniformly propagating solitary waves, which at high momentum can be viewed as Skyrmion-antiSkyrmion pairs. We study these solitary waves for a range of interaction regimes and show that for experimentally relevant cases there is a transition to spatially extended spin-wave states at low momentum. We explain how this can be understood by analogy to the 2D ferromagnet. Finally, we discuss how such solitary waves can be generated and studied in experiment.

Kazuya Fujimoto (Osaka, Japan)

Counterflow instability and turbulent state for spin-1 spinor Bose-Einstein condensates

Quantum turbulence (QT) in atomic Bose-Einstein condensates (BECs) is experimentally [1] and theoretically [2] studied, but most of studies focus on one-component atomic BECs. Recently, theoretical study for the quantum turbulence of two-component BECs is performed [3], but QT of spinor BECs has never been studied. We are interested in the behavior for the turbulent state of the spinor BECs, for example, the dynamics of spin and superfluid velocity in the turbulent state. Our main target is to find that, in the turbulent state, the spin of spinor BECs shows the self-similar structure like vortices in QT of one-component BECs and what character the coupled turbulence of the spin and the superfluid velocity has. In our presentation, we mainly show the results characteristic to the spin. We theoretically study counter flow instability for the spin-1 spinor BEC, which makes the turbulent state, by means of the Bogoliubov-de Genunes (BdG) and the Gross-Pitaevskii (GP) equations. Our study treats the counterflow between $m = 1$ and $m =$ -1 components in a uniform two-dimensional system as shown in Fig. 1, where m is a magnetic quantum number. We find that the instability induces density modulation corresponding to the most unstable wave number of the dispersion relation obtained by the BdG equation, leading to the exchange of the particles between the different components and the grow of the amplitude of the spin density. As time sufficiently goes by, the spin density and the probability density function of the spin show that the spin randomly points to various direction as shown in Fig. 2, which means the appearance of uniform and isotropic spin turbulent state. We numerically calculate the spectrum of the spin dependent interaction energy in the turbulence state, finding that the spectrum shows the power law in the case for ferromagnetic interaction. We present the details of the counterflow instability and the spin turbulent state for the spin-1 spinor BEC.

References :

[1] E. A. L. Henn, J. A. Seman, G. Roati, K. M. Magalhaes, and V. S. Bagnato, Phys. Rev. Lett. 103, 045301 (2009).

[2] M. Kobayashi and M. Tsubota, Phys. Rev, A 76, 045603 (2007).

[3] H. Takeuchi, S. Ishino and M. Tsubota, Phys. Rev. Lett. 105, 205301 (2010).

Friday December 2nd

9:00 - 9:45

Jean Dalibard (Paris, France)

Artificial gauge potentials for neutral atoms

The simulation of condensed matter systems is certainly one of the most appealing perspectives opened by the recent developments in the physics of cold atomic gases. Among the large variety of quantum collective phenomena that one hopes to address with atomic vapours, magnetism is one of the richest. However the quest for the simulation of magnetism immediately raises a challenging question: how can a system of neutral atoms behave as an assembly of charged particles in a magnetic field ?

The talk will review some promising approaches to answer this question both in a bulk system and in an optical lattice. A first already well-explored path is to rotate the gas and take advantage of the similarity between Lorentz and Coriolis forces. A second possibility is based on the Berry's phase that an atom accumulates when it follows adiabatically one of its internal levels during its motion in a well-chosen energy landscape. In both cases one of the main challenges is to produce atomic states that would be analogous of those characteristic of fractional quantum Hall physics.

For a review, see e.g. Dalibard J., Gerbier F., Juzeliunas G., and Öhberg P., arXiv:1008.5378, to appear in Rev. Mod. Phys.

10:00 - 10:45

Francis Nier (Rennes, France)

Artificial gauge and adiabatic ansatz for Bose-Einstein condensates

About two years ago, in a workshop of this ANR project, Jean Dalibard adressed the question whether the adiabatic ansatz could be mathematically analyzed, within the modeling of simulated gauge fields for Bose-Einstein condensates. We have considered this problem with A. Aftalion. Surprisingly the main difficulties do not come from the nonlinear part but from the linear Hamiltonian. Actually, deriving rigorously the asymptotic model, guessed by the physicists, and showing that this asymptotic analysis contains the information about the presence of vortices, requires to reconsider carefully previous mathematical works about the adiabatic limit. Although the estimates are not sharp, this asymptotic analysis provides a good insight about the validity of the asymptotic picture, with respect to the size of the many parameters initially put in the complete model.

11:15 - 12:00

Nigel Cooper (Cambridge, UK)

Optical Flux Lattices for Ultracold Atomic Gases

I will describe how simple laser configurations can give rise to "optical flux lattices", in which optically dressed atoms experience a periodic effective magnetic flux with high mean density. These potentials lead to narrow energy bands with nonzero Chern numbers. Optical flux lattices will greatly facilitate the achievement of the quantum Hall regime for ultracold atomic gases.

13:30 - 14:15

Gilles Montambaux (Orsay, France)

Berry phase, Dirac points in two-dimensional crystals

We consider several examples of 2D crystals where the low energy properties can be described by a 2 x 2 Hamiltonian with a spectrum exhibiting several Dirac cones. These cones are characterized by a linear dispersion relation, but more importantly by a topological number, or "charge", related to a Berry phase associated to the spinorial structure of the wave function. For example, the graphene spectrum has a pair of Dirac cones with opposite Berry phases ($\pm\pi$). Such a situation is also encountered in a square optical lattice of cold atoms in the presence of artificial gauge field. We study under which conditions these Dirac cones can be manipulated, created or suppressed, through a modification of band parameters, under the condition of conservation of the total "charge".

At the transition where two Dirac points with opposite "charges" merge, the spectrum has the remarkable property to be linear in one direction and quadratic in the other direction (semi-Dirac point). We derive a universal Hamiltonian that describes the vicinity of the transition, characterized by three parameters, a mass, a velocity and a driving parameter $\triangle$ whose values are related to the band parameters of any 2D crystal with time-reversal and inversion symmetries. This model describes continuously the coupling between valleys associated with the two Dirac points, when approaching the transition $\triangle$ =0. A pair of Dirac points with same charge can also merge into a single point with double charge 2π, characterized by a gapless quadratic spectrum. This merging belongs to a different universality class, and the scaling behavior of the Landau levels is different from the previous case.

I will discuss some aspects of the physics of Dirac points in various models of two-dimensional crystals. Those include the honeycomb lattice, the square lattice and the dice (T3) lattice in a magnetic field. The corresponding electronic spectra exhibit Dirac points with various topological properties.

References :

[1] A universal Hamiltonian for the motion and the merging of Dirac cones in a two-dimensional crystal, G. Montambaux, F. Piéchon, J.-N. Fuchs and M.O. Goerbig, Eur. Phys. J. B 72, 509 (2009)

[2] Merging of Dirac points in a two-dimensional crystal, G. Montambaux, F. Piéchon, J.-N. Fuchs and M.O. Goerbig, Phys. Rev. B 80, 153412 (2009)

[3] A semi-Dirac point in the Hofstadter spectrum P. Delplace and G. Montambaux, Phys. Rev. B 82, 035438 (2010)

14:30 - 15:15

Bernard Helffer (Orsay, France)

On Harper's equation : Historic and new questions

If the first mathematical results were obtained more than 30 years ago with the interpretation of the celebrated Hofstadter butterfly, more recent experiments in Bose-Einstein theory suggest new questions. I will preseent a survey on sometime old results (Helffer-Sjöstrand, Avila,...) and also discuss more recent questions with generalized butterflys (Hou, Royo-Letellier)